السنة 2 بكالوريا علوم تجريبية	الحساب التكاملي	سلسلة 2

تمرين 1 : نعتبر التكاملين : $I=\int_0^{\frac{®}{2}} \frac{\cos(x)}{1+2\sin(x)}dx$ و $J=\int_0^{\frac{®}{2}} \frac{\sin(2x)}{1+2\sin(x)}dx$

1. $$I=\int_0^{\frac{®}{2}} \frac{\cos(x)}{1+2\sin(x)}dx=\int_0^{\frac{®}{2}} \frac{1}{2}\frac{2\cos(x)}{1+2\sin(x)}dx=\frac{1}{2}\int_0^{\frac{®}{2}} \frac{(1+2\sin(x))'}{1+2\sin(x)}dx=\frac{1}{2}\left[\ln(|1+2\sin(x)|)\right]_0^{\frac{®}{2}}=\frac{\ln(3)}{2}$$

2. $$I+J=\int_0^{\frac{®}{2}} \frac{\cos(x)}{1+2\sin(x)}dx+\int_0^{\frac{®}{2}} \frac{\sin(2x)}{1+2\sin(x)}dx=\int_0^{\frac{®}{2}} \frac{\cos(x)+\sin(2x)}{1+2\sin(x)}dx$$
$$I+J=\int_0^{\frac{®}{2}} \frac{\cos(x)+2\sin(x)\cos(x)}{1+2\sin(x)}dx=\int_0^{\frac{®}{2}} \frac{\cos(x)(1+2\sin(x))}{1+2\sin(x)}dx=\int_0^{\frac{®}{2}} \cos(x)\,dx=\left[\sin(x)\right]_0^{\frac{®}{2}}=1$$

3. $$J=(I+J)-I=1-\frac{\ln(3)}{2}$$

تمرين 2 : $I=\int_0^{\frac{®}{2}} \cos^3(x)dx$ و $J=\int_0^{\frac{®}{2}} \sin^2(x)\cos(x)\,dx$

1. $$I+J=\int_0^{\frac{®}{2}} \cos^3(x)dx+\int_0^{\frac{®}{2}} \sin^2(x)\cos(x)\,dx=\int_0^{\frac{®}{2}} \cos^3(x)+\sin^2(x)\cos(x)\,dx$$
$$I+J=\int_0^{\frac{®}{2}} \cos(x)\left(\cos^2(x)+\sin^2(x)\right)dx=\int_0^{\frac{®}{2}} \cos(x)\,dx=\left[\sin(x)\right]_0^{\frac{®}{2}}=1$$

2. $$J=\int_0^{\frac{®}{2}} \sin^2(x)\cos(x)\,dx=\int_0^{\frac{®}{2}} \sin^2(x)(\sin(x))'\,dx=\left[\frac{1}{3}\sin^3(x)\right]_0^{\frac{®}{2}}=\frac{1}{3}$$

3. $$I=(I+J)-J=1-\frac{1}{3}=\frac{2}{3}$$

تمرين 3 :

نضع : $\begin{cases}u(x)=x\\ v(x)=\sin(x)\end{cases}$ منه : $\begin{cases}u'(x)=1\\ v'(x)=\cos(x)\end{cases}$

$$\int_0^{\frac{®}{2}} x\cos(x)dx=\int_0^{\frac{®}{2}} u(x)\,v'(x)dx=\left[u(x)v(x)\right]_0^{\frac{®}{2}}-\int_0^{\frac{®}{2}} u'(x)v(x)dx=\left[x\sin(x)\right]_0^{\frac{®}{2}}-\int_0^{\frac{®}{2}} \sin(x)dx$$
$$\int_0^{\frac{®}{2}} x\cos(x)dx=\left[x\sin(x)\right]_0^{\frac{®}{2}}-\left[\cos(x)\right]_0^{\frac{®}{2}}=\left(\frac{®}{2}-0\right)-(0-1)=\frac{®}{2}+1$$

نضع : $\begin{cases}u(x)=3+2x\\ v(x)=e^x\end{cases}$ منه : $\begin{cases}u'(x)=2\\ v'(x)=e^x\end{cases}$

$$\int_0^1 (3+2x)e^x\,dx=\int_0^1 u(x)v'(x)dx=\left[u(x)v(x)\right]_0^1-\int_0^1 u'(x)v(x)e^x\,dx=\left[(3+2x)e^x\right]_0^1-\int_0^1 2e^x\,dx$$
$$=\left[(3+2x)e^x\right]_0^1-\left[2e^x\right]_0^1=(5e-3)-(2e-2)=3e-1$$

نضع : $\begin{cases}u(x)=x\\ v(x)=\ln x\end{cases}$ منه : $\begin{cases}u'(x)=1\\ v'(x)=\frac{1}{x}\end{cases}$

$$\int_1^2 \ln(x)\,dx=\int_1^2 u'(x)v(x)\,dx=\left[u(x)v(x)\right]_1^2-\int_1^2 u(x)v'(x)dx=\left[x\ln(x)\right]_1^2-\int_1^2 x\times\frac{1}{x}dx$$
$$\int_1^2 \ln(x)\,dx=\left[x\ln(x)\right]_1^2-\int_1^2 1dx=\left[x\ln(x)\right]_1^2-\left[x\right]_1^2=(2\ln(2)-0)-(2-1)=2\ln(2)-1$$

نضع : $\begin{cases} u(x)=\dfrac{-1}{x} \\ v(x)=\ln x \end{cases}$ منه : $\begin{cases} u'(x)=\dfrac{1}{x^2} \\ v'(x)=\dfrac{1}{x} \end{cases}$

$$\int_1^e \frac{\ln(x)}{x^2}dx=\int_1^e u'(x)v(x)dx=[u(x)v(x)]_1^e-\int_1^e u(x)v'(x)dx=\left[\frac{-\ln(x)}{x}\right]_1^e-\int_1^e \frac{-1}{x}\times\frac{1}{x}dx$$

$$\int_1^e \frac{\ln(x)}{x^2}dx=\left[\frac{-\ln(x)}{x}\right]_1^e+\int_1^e \frac{1}{x^2}dx=\left[\frac{-\ln(x)}{x}\right]_1^e+\left[\frac{-1}{x}\right]_1^e=\left(\frac{-1}{e}+0\right)+\left(\frac{-1}{e}+1\right)=\frac{-2}{e}+1$$

تمرين 4 : $I=\int_0^{\frac{\pi}{2}} \cos(x)e^x\,dx$ ، $J=\int_0^{\frac{\pi}{2}} \sin(x)\,e^x\,dx$

1

$$J=\int_0^{\frac{\pi}{2}} \sin(x)\,e^x\,dx=\int_0^{\frac{\pi}{2}} \sin(x)\left(e^x\right)'dx=\left[\sin(x)e^x\right]_0^{\frac{\pi}{2}}-\int_0^{\frac{\pi}{2}} (\sin(x))'\,e^x\,dx$$

$$J=e^{\frac{\pi}{2}}-\int_0^{\frac{\pi}{2}} \cos(x)\,e^x\,dx=e^{\frac{\pi}{2}}-I$$

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$$I=\int_0^{\frac{\pi}{2}} \cos(x)\,e^x\,dx=\int_0^{\frac{\pi}{2}} \cos(x)\left(e^x\right)'dx=\left[\cos(x)e^x\right]_0^{\frac{\pi}{2}}-\int_0^{\frac{\pi}{2}} (\cos(x))'\,e^x\,dx$$

$$I=-1-\int_0^{\frac{\pi}{2}} -\sin(x)\,e^x\,dx=-1+J$$

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$$\begin{cases} J=e^{\frac{\pi}{2}}-I \\ I=-1+J \end{cases}\Rightarrow\begin{cases} J=e^{\frac{\pi}{2}}-I \\ I=-1+e^{\frac{\pi}{2}}-I \end{cases}\Rightarrow\begin{cases} J=e^{\frac{\pi}{2}}-\dfrac{-1+e^{\frac{\pi}{2}}}{2} \\ I=\dfrac{-1+e^{\frac{\pi}{2}}}{2} \end{cases}\Rightarrow\begin{cases} J=\dfrac{1+e^{\frac{\pi}{2}}}{2} \\ I=\dfrac{-1+e^{\frac{\pi}{2}}}{2} \end{cases}$$

تمرين 5 :

1

$$I=\int_0^1 x^2e^x\,dx=\int_0^1 u(x)v'(x)\,dx$$

نضع : $\begin{cases} u(x)=x^2 \\ v(x)=e^x \end{cases}$ منه : $\begin{cases} u'(x)=2x \\ v'(x)=e^x \end{cases}$ ، منه :

$$I=[u(x)v(x)]_0^1-\int_0^1 u'(x)v(x)\,dx$$

$$I=\left[x^2e^x\right]_0^1-\int_0^1 2xe^x\,dx=e-\int_0^1 2xe^x\,dx$$

مرة أخرى نضع : $\begin{cases} u(x)=2x \\ v(x)=e^x \end{cases}$ منه : $\begin{cases} u'(x)=2 \\ v'(x)=e^x \end{cases}$ ، منه :

$$\int_0^1 2xe^x\,dx=\int_0^1 u(x)v'(x)\,dx=[u(x)v(x)]_0^1-\int_0^1 u'(x)v(x)\,dx$$

$$\int_0^1 2xe^x\,dx=\left[2xe^x\right]_0^1-\int_0^1 2\,e^x\,dx=2e-\left[2e^x\right]_0^1=2e-(2e-2)=2$$

، بالتالي : $I=e-2$

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أ)

$$F'(x)=x^2e^x \iff (2x+a)e^x+\left(x^2+ax+b\right)e^x=x^2e^x$$

$$F'(x)=x^2e^x \iff e^x\left(2x+a+x^2+ax+b\right)=x^2e^x$$

$$F'(x)=x^2e^x \iff 2x+a+x^2+ax+b=x^2$$

$$F'(x)=x^2e^x \iff (2+a)x+a+b=0$$

$$F'(x)=x^2e^x \iff \begin{cases} 2+a=0 \\ a+b=0 \end{cases}\iff\begin{cases} a=-2 \\ b=2 \end{cases}$$

ب)

$$I=\int_0^1 x^2e^x\,dx=\left[\left(x^2-2x+2\right)e^x\right]_0^1=e-2$$

تمرين 6 : $I_n = \int_1^e t^n \ln(t)\, dt \quad n \in IN^*$	
نضع : $\begin{cases} u(t) = \frac{t^{n+1}}{n+1} \\ v(t) = \ln(t) \end{cases}$ منه : $\begin{cases} u'(t) = t^n \\ v'(t) = \frac{1}{t} \end{cases}$ ، منه : $I_n = \int_1^e t^n \ln(t)\, dt = \int_1^e u'(t)v(t)\, dt = [u(t)v(t)]_1^e - \int_1^e u(t)v'(t)\, dt = \left[\frac{t^{n+1}}{n+1}\ln(t)\right]_1^e - \int_1^e \frac{t^{n+1}}{n+1} \times \frac{1}{t}\, dt$ $I_n = \frac{e^{n+1}}{n+1} - \frac{1}{n+1}\int_1^e t^n\, dt = \frac{e^{n+1}}{n+1} - \frac{1}{n+1}\left[\frac{t^{n+1}}{n+1}\right]_1^e = \frac{e^{n+1}}{n+1} - \frac{1}{n+1}\left(\frac{e^{n+1}}{n+1} - \frac{1}{n+1}\right)$ $I_n = \frac{e^{n+1}}{n+1}\left(1 - \frac{1}{n+1}\right) + \frac{1}{(n+1)^2} = \frac{e^{n+1}}{n+1} \times \frac{n}{n+1} + \frac{1}{(n+1)^2}$	1
نعلم أن : $\lim_{n\to+\infty} \frac{e^{n+1}}{n+1} = +\infty$ و $\lim_{n\to+\infty} \frac{n}{n+1} = \lim_{n\to+\infty} \frac{n}{n} = 1$ و $\lim_{n\to+\infty} \frac{1}{(n+1)^2} = 0$ ، إذن : $\lim_{n\to+\infty} I_n = +\infty$	2