

سلسلة 1 للجذور المربعة



تمرين 1 :

(1) حدد جميع جذور العدد 81 ؟

(2) أحسب مربع الأعداد التالية :

$$A = 9 ; B = \sqrt{5} ; C = \sqrt{3,7} ; D = \frac{\sqrt{7}}{5} ; E = -3\sqrt{2} ; F = \sqrt{(-3)^4}$$

(3) حدد الأعداد التي لها جذور مربعة وأحسبها ؟

$$A = 64 ; B = -81 ; C = (-5)^2 ; D = (-5)^3$$

تمرين 2 :

أحسب مايلي :

$$A = \sqrt{16} ; B = \sqrt{\pi^2} ; C = \sqrt{5,3^2} ; D = \sqrt{17^2} ; E = \sqrt{0,36}$$

$$F = (-2\sqrt{5})^2 ; G = \frac{\sqrt{50}}{\sqrt{2}} ; H = \sqrt{\left(\frac{2}{3}\right)^2} ; I = \sqrt{\frac{4}{9}}$$

$$J = \sqrt{2} \times \sqrt{8} ; K = \sqrt{2} \times \sqrt{3} \times \sqrt{6} ; L = \left[3 \left(\sqrt{\frac{5}{3}}\right)^2\right]^2$$

تمرين 3 :

أكتب على شكل $a\sqrt{b}$ الأعداد التالية :

$$\begin{array}{llll} A = \sqrt{18} & ; & B = \sqrt{20} & ; & C = \sqrt{24} & ; & D = \sqrt{32} \\ E = \sqrt{45} & ; & F = \sqrt{50} & ; & G = \sqrt{63} & ; & H = \sqrt{75} \\ I = \sqrt{99} & ; & J = \sqrt{363} & ; & K = \sqrt{700} & ; & L = \sqrt{1000} \end{array}$$

تمرين 4 :

بسّط وأحسب ما يلي :

$$F = \sqrt{3 + \sqrt{9}}$$

$$G = \sqrt{3 \times \sqrt{9}}$$

$$H = \left(\sqrt{3 + \sqrt{5}}\right)^2$$

$$I = \sqrt{5 \times 9 + 4}$$

$$J = \sqrt{8} \times \sqrt{50} \times \sqrt{18}$$

$$A = \sqrt{2} + \sqrt{2} + \sqrt{2} + \sqrt{2}$$

$$B = \sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{2}$$

$$C = 5\sqrt{3} + 6 - 3\sqrt{3} - 2$$

$$D = 4\sqrt{5} - 3\sqrt{2} + 3\sqrt{5} + \sqrt{2}$$

$$E = 3\sqrt{2} \times 4\sqrt{5} - 2\sqrt{5} \times \frac{3}{2}\sqrt{7}$$

تمرين 5 :

أحسب :

$$A = \sqrt{25} + \sqrt{16} ; B = \sqrt{25} - \sqrt{16} ; C = \sqrt{25} \times \sqrt{16} ; D = \sqrt{\frac{25}{16}}$$

تمرين 6 :

حل في \mathbb{R} المعادلات التالية :

$$x^2 = 5 ; 3x^2 + 5 = 5 ; x^2 - 8 = 1 ; -3x^2 = 12$$

سلسلة 1 للجذور المربعة



تمرين 1 :

(1) حدد جميع جذور العدد 81 ؟

$$\sqrt{81} = \sqrt{9^2} = 9 \quad \text{للعدد 81 جذر واحد هو 9 لأن}$$

(2) أحسب مربع الأعداد التالية :

$$A = 9^2 = 81 \quad ; \quad B = \sqrt{5}^2 = 5 \quad ; \quad C = \sqrt{3,7}^2 = 3,7$$

$$D = (-3\sqrt{2})^2 = (-3)^2 \times \sqrt{2}^2 = 9 \times 2 = 18 \quad ; \quad E = \left(\frac{\sqrt{7}}{5}\right)^2 = \frac{7}{25}$$

$$F = \left(\sqrt{(-3)^4}\right)^2 = (-3)^4 = 3^4$$

(3) جميع الأعداد الموجبة لها جذر مربع واحد والأعداد السالبة ليس لها جذر مربع إذن الأعداد

التي لها جذر مربع هي :

$$A = \sqrt{64} = 8 \quad ; \quad C = (-5)^2 = 5^2 = 25$$

تمرين 2 :

أحسب مايلي :

$$H = \sqrt{\left(\frac{2}{3}\right)^2} = \frac{2}{3}$$

$$I = \sqrt{\frac{4}{9}} = \sqrt{\left(\frac{2}{3}\right)^2} = \frac{2}{3}$$

$$J = \sqrt{2} \times \sqrt{8} = \sqrt{2 \times 8} = \sqrt{16} = 4$$

$$K = \sqrt{2} \times \sqrt{3} \times \sqrt{6} = \sqrt{6} \times \sqrt{6} = \sqrt{6^2} = 6$$

$$L = \left[3 \left(\sqrt{\frac{5}{3}}\right)^2\right]^2 = \left(3 \times \frac{5}{3}\right)^2 = 25$$

$$A = \sqrt{16} = \sqrt{4^2} = 4$$

$$B = \sqrt{\pi^2} = \pi$$

$$C = \sqrt{5,3^2} = 5,3$$

$$D = \sqrt{17^2} = 17$$

$$E = \sqrt{0,36} = \sqrt{0,6^2} = 0,6$$

$$F = (-2\sqrt{5})^2 = (-2)^2 \times \sqrt{5}^2 = 4 \times 5 = 20$$

$$G = \frac{\sqrt{50}}{\sqrt{2}} = \frac{\sqrt{2 \times 25}}{\sqrt{2}} = 5$$

تمرين 3 :

أكتب على شكل $a\sqrt{b}$ الأعداد التالية :

$$G = \sqrt{63} = \sqrt{9 \times 7} = 3\sqrt{7}$$

$$H = \sqrt{75} = \sqrt{25 \times 3} = 5\sqrt{3}$$

$$I = \sqrt{99} = \sqrt{9 \times 11} = 3\sqrt{11}$$

$$J = \sqrt{363} = \sqrt{121 \times 3} = 11\sqrt{3}$$

$$K = \sqrt{700} = \sqrt{100 \times 7} = 10\sqrt{7}$$

$$L = \sqrt{1000} = \sqrt{100 \times 10} = 10\sqrt{10}$$

$$A = \sqrt{18} = \sqrt{9 \times 2} = 3\sqrt{2}$$

$$B = \sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5}$$

$$C = \sqrt{24} = \sqrt{4 \times 6} = 2\sqrt{6}$$

$$D = \sqrt{32} = \sqrt{16 \times 2} = 4\sqrt{2}$$

$$E = \sqrt{45} = \sqrt{9 \times 5} = 3\sqrt{5}$$

$$F = \sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2}$$

تمرين 4 : بسط وأحسب ما يلي :

$$A = \sqrt{2} + \sqrt{2} + \sqrt{2} + \sqrt{2} = 4\sqrt{2}$$

$$B = \sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{2} = \sqrt{2^4} = (\sqrt{2^2})^2 = 2^2 = 4$$

$$C = 5\sqrt{3} + 6 - 3\sqrt{3} - 2 = 5\sqrt{3} - 3\sqrt{3} + 6 - 2 = 2\sqrt{3} + 4$$

$$D = 4\sqrt{5} - 3\sqrt{2} + 3\sqrt{5} + \sqrt{2} = 4\sqrt{5} + 3\sqrt{5} - 3\sqrt{2} + \sqrt{2} = 7\sqrt{5} - 2\sqrt{2}$$

$$E = 3\sqrt{2} \times 4\sqrt{5} - 2\sqrt{5} \times \frac{3}{2}\sqrt{7} = 3 \times 4\sqrt{2 \times 5} - 2 \times \frac{3}{2}\sqrt{5 \times 7} \\ = 12\sqrt{10} - 3\sqrt{35}$$

$$F = \sqrt{3 + \sqrt{9}} = \sqrt{3 + 3} = \sqrt{6}$$

$$G = \sqrt{3 \times \sqrt{9}} = \sqrt{3 \times 3} = \sqrt{3^2} = 3$$

$$H = (\sqrt{3 + \sqrt{5}})^2 = 3 + \sqrt{5}$$

$$I = \sqrt{5 \times 9 + 4} = \sqrt{45 + 4} = \sqrt{49} = 7$$

$$J = \sqrt{8} \times \sqrt{50} \times \sqrt{18} = \sqrt{4 \times 2} \times \sqrt{25 \times 2} \times \sqrt{9 \times 2}$$

$$= 2\sqrt{2} \times 5\sqrt{2} \times 3\sqrt{2} = 2 \times 5 \times 3\sqrt{2^2} \times \sqrt{2} = 2 \times 5 \times 3 \times 2\sqrt{2} = 60\sqrt{2}$$

تمرين 5 :

أحسب :

$$A = \sqrt{25} + \sqrt{16} = 5 + 4 = 9 \quad ; \quad B = \sqrt{25} - \sqrt{16} = 5 - 4 = 1$$

$$C = \sqrt{25} \times \sqrt{16} = 5 \times 4 = 20 \quad ; \quad D = \sqrt{\frac{25}{16}} = \sqrt{\left(\frac{5}{4}\right)^2} = \frac{5}{4}$$

تمرين 6 :

حل في \mathbb{R} المعادلات التالية :

$$x^2 - 8 = 1$$

$$x^2 - 9 = 0$$

$$x^2 - 3^2 = 0$$

$$(x - 3)(x + 3) = 0$$

$$(x - 3) = 0 \quad \text{أو} \quad (x + 3) = 0$$

$$x = 3 \quad \text{أو} \quad x = -3$$

المعادلة تقبل حلين هما : 3 و -3

$$-3x^2 = 12$$

$$x^2 = \frac{12}{-3}$$

$$x^2 = -4$$

بما أن $x^2 > 0$ إذن المعادلة لا تقبل حل

$$x^2 = 5 \Leftrightarrow x^2 - 5 = 0$$

$$x^2 - \sqrt{5}^2 = 0$$

$$(x - \sqrt{5})(x + \sqrt{5}) = 0$$

$$x - \sqrt{5} = 0 \quad \text{أو} \quad x + \sqrt{5} = 0$$

$$x = \sqrt{5} \quad \text{أو} \quad x = -\sqrt{5}$$

المعادلة تقبل حلين هما : $\sqrt{5}$ و $-\sqrt{5}$

$$3x^2 + 5 = 5 \Leftrightarrow 3x^2 = 5 - 5$$

$$x^2 = \frac{0}{3} \Leftrightarrow x^2 = 0$$

$$x = 0 \quad \text{إذن}$$

إذن المعادلة تقبل حل وحيد هو 0

سلسلة 2 للجذور المربعة



تمرين 1:

أجب بصحيح أو خطأ:

$$\sqrt{a} + \sqrt{a} + \sqrt{a} = 3\sqrt{a}$$

$$\sqrt{a} \times \sqrt{a} \times \sqrt{a} = 3\sqrt{a}$$

$$\sqrt{(-7)^2} = -7$$

$$x^2 = -5 \text{ يوجد } x \text{ بحيث}$$

$$\sqrt{a+b} = \sqrt{a} + \sqrt{b}$$

$$\sqrt{a} - \sqrt{b} = \sqrt{a-b}$$

$$\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$$

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

$$\sqrt{36+64} = \sqrt{36} + \sqrt{64} = 6+8=14$$

$$\sqrt{36+64} = \sqrt{100} = \sqrt{10^2} = 10$$

$$\sqrt{100-36} = \sqrt{64} = \sqrt{8^2} = 8$$

$$\sqrt{100-36} = \sqrt{100} - \sqrt{36} = 10-6=4$$

$$\sqrt{16} + \sqrt{9} = \sqrt{16+9} = \sqrt{25} = 5$$

$$\sqrt{16} + \sqrt{9} = 4+3=7$$

$$\sqrt{5} \times 2\sqrt{3} = \sqrt{5 \times 2 \times 3} = \sqrt{30}$$

$$\sqrt{5} \times 2\sqrt{3} = 2\sqrt{5 \times 3} = 2\sqrt{15}$$

$$\sqrt{\sqrt{9}+6} \times \sqrt{9} = \sqrt{\sqrt{9}} + \sqrt{6} \times \sqrt{9} = \sqrt{3} + \sqrt{54} = \sqrt{57}$$

$$\sqrt{\sqrt{9}+6} \times \sqrt{9} = \sqrt{3+6} \times \sqrt{9} = \sqrt{9} \times \sqrt{9} = \sqrt{9^2} = 9$$

تمرين 2:

أحسب:

$$E = \sqrt{8^2 + 6^2}$$

$$F = \sqrt{3} + \sqrt{81} - \sqrt{9}$$

$$G = \sqrt{3^2 \times 2^2}$$

$$I = \frac{\sqrt{25} + \sqrt{16}}{\sqrt{9}}$$

$$A = \sqrt{\sqrt{10^4}}$$

$$B = \sqrt{\sqrt{25^2}}$$

$$C = \sqrt{\sqrt{16}}$$

$$D = 5\sqrt{81}$$

تمرين 3 :

أتمم مايلي :

$$A = \sqrt{2 \times \dots} = 10 \quad ; \quad B = \sqrt{40 + \dots} = 7 \quad ; \quad C = \sqrt{99} = \dots \sqrt{11}$$

$$D = \sqrt{\dots} = 5\sqrt{2} \quad ; \quad E = \sqrt{10000} = \dots \quad ; \quad F = \sqrt{9 + \dots} = 3 + 4$$

تمرين 4 :

بسط مايلي :

$$A = \sqrt{a^2} - \sqrt{b^2} - a + b$$

$$B = \sqrt{a^4 b^2} \times \sqrt{a^3 b} \times \sqrt{ab^3}$$

$$C = \sqrt{2 + \sqrt{2}} \times \sqrt{2 + \sqrt{2 + \sqrt{2}}} \times \sqrt{2 - \sqrt{2 + \sqrt{2}}}$$

تمرين 5 :

أكتب الأعداد التالية على شكل $a\sqrt{b}$:

$$E = -2\sqrt{12} - \sqrt{75} + \sqrt{300}$$

$$F = 8\sqrt{5} - \sqrt{500} + 4\sqrt{45}$$

$$G = 4\sqrt{24} + 6\sqrt{6} - 2\sqrt{600}$$

$$H = \sqrt{44} + \sqrt{99} + 6\sqrt{11}$$

$$A = 2\sqrt{5} + 5\sqrt{20} =$$

$$B = \sqrt{45} - \sqrt{5} + 3\sqrt{20}$$

$$C = \sqrt{63} + \sqrt{28} - \sqrt{700}$$

$$D = \sqrt{50} + 3\sqrt{18} - \sqrt{32}$$

تمرين 6 :

احذف الجذر من المقام :

$$\frac{5}{\sqrt{3}} \quad ; \quad \frac{-5}{2\sqrt{3}} \quad ; \quad \frac{2\sqrt{5}}{\sqrt{3}} \quad ; \quad \frac{1}{2-\sqrt{3}} \quad ; \quad \frac{2}{\sqrt{5}+3} \quad ; \quad \frac{2\sqrt{3}}{\sqrt{5}+1} \quad ;$$

$$\frac{\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}} \quad ; \quad \frac{1}{1+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{7}} + \frac{1}{\sqrt{7}+\sqrt{9}}$$

تمرين 7 :

حل في \mathbb{R} المعادلات التالية :

$$\frac{6+2x^2}{3} - x^2 = 2$$

$$9x^2 = 4x^2$$

$$x^2 + 3 = 0$$

$$x^2 - 36 = 0$$

$$5 - x^2 = -4$$

$$\frac{5}{12}x^2 - \frac{5}{3} = 0$$

حل السلسلة 2 للجذور المربعة



تمرين 1:

أجب بصحيح أو خطأ:

$$\sqrt{a} + \sqrt{a} + \sqrt{a} = 3\sqrt{a}$$

صحيح

$$\sqrt{a} \times \sqrt{a} \times \sqrt{a} = 3\sqrt{a}$$

خطأ

$$\sqrt{(-7)^2} = -7$$

خطأ

يوجد x بحيث $x^2 = -5$

خطأ

$$\sqrt{36 + 64} = \sqrt{36} + \sqrt{64} = 6 + 8 = 14$$

خطأ

$$\sqrt{36 + 64} = \sqrt{100} = \sqrt{10^2} = 10$$

صحيح

$$\sqrt{100 - 36} = \sqrt{64} = \sqrt{8^2} = 8$$

صحيح

$$\sqrt{100 - 36} = \sqrt{100} - \sqrt{36} = 10 - 6 = 4.$$

خطأ

$$\sqrt{16} + \sqrt{9} = \sqrt{16 + 9} = \sqrt{25} = 5$$

خطأ

$$\sqrt{16} + \sqrt{9} = 4 + 3 = 7$$

صحيح

$$\sqrt{5} \times 2\sqrt{3} = \sqrt{5 \times 2 \times 3} = \sqrt{30}$$

خطأ

$$\sqrt{5} \times 2\sqrt{3} = 2\sqrt{5 \times 3} = 2\sqrt{15}$$

صحيح

$$\sqrt{\sqrt{9} + 6} \times \sqrt{9} = \sqrt{\sqrt{9} + 6} \times \sqrt{9} = \sqrt{3} + \sqrt{54} = \sqrt{57}$$

خطأ

$$\sqrt{\sqrt{9} + 6} \times \sqrt{9} = \sqrt{3 + 6} \times \sqrt{9} = \sqrt{9} \times \sqrt{9} = \sqrt{9^2} = 9$$

صحيح

تمرين 2:

أحسب:

$$A = \sqrt{\sqrt{10^4}} = \sqrt{(\sqrt{10^2})^2} = 10$$

$$B = \sqrt{\sqrt{25^2}} = \sqrt{\sqrt{5^2}^2} = 5$$

$$C = \sqrt{\sqrt{16}} = \sqrt{\sqrt{4^2}} = \sqrt{4} = 2$$

$$D = 5\sqrt{81} = 5\sqrt{9^2} = 5 \times 9 = 45$$

$$E = \sqrt{8^2 + 6^2} = \sqrt{64 + 36} = \sqrt{100} = \sqrt{10^2} = 10$$

$$F = \sqrt{3} + \sqrt{81} - \sqrt{9} = \sqrt{3} + 9 - 3 = \sqrt{3} + 6$$

$$G = \sqrt{3^2 \times 2^2} = \sqrt{9 \times 4} = \sqrt{36} = 6$$

$$I = \frac{\sqrt{25} + \sqrt{16}}{\sqrt{9}} = \frac{5+4}{3} = \frac{9}{3} = 3$$

تمرين 3 :

أتمم مايلي :

$$A = \sqrt{2 \times 25} = 10 \quad ; \quad B = \sqrt{40 + 9} = 7 \quad ; \quad C = \sqrt{99} = 3\sqrt{11}$$

$$D = \sqrt{100} = 5\sqrt{2} \quad ; \quad E = \sqrt{10000} = 100 \quad ; \quad F = \sqrt{9 + 40} = 3 + 4$$

تمرين 4 :

بسّط مايلي :

$$A = \sqrt{a^2} - \sqrt{b^2} - a + b = a - b - a + b = a - a - b + b = 0$$

$$B = \sqrt{a^4 b^2} \times \sqrt{a^3 b} \times \sqrt{ab^3} = \sqrt{a^4 b^2 \times a^3 b \times ab^3} = \sqrt{a^{4+3+1} \times b^{2+1+3}}$$
$$= \sqrt{a^6 b^6} = \sqrt{(ab)^6} = \left(\sqrt{(ab)^2}\right)^3 = (ab)^3$$

$$C = \sqrt{2 + \sqrt{2}} \times \sqrt{2 + \sqrt{2 + \sqrt{2}}} \times \sqrt{2 - \sqrt{2 + \sqrt{2}}}$$
$$= \sqrt{2 + \sqrt{2}} \times \sqrt{(2 + \sqrt{2 + \sqrt{2}}) \times (2 - \sqrt{2 + \sqrt{2}})}$$
$$= \sqrt{2 + \sqrt{2}} \times \sqrt{2^2 - (\sqrt{2 + \sqrt{2}})^2} = \sqrt{2 + \sqrt{2}} \times \sqrt{4 - (2 + \sqrt{2})}$$
$$= \sqrt{2 + \sqrt{2}} \times \sqrt{4 - 2 - \sqrt{2}} = \sqrt{2 + \sqrt{2}} \times \sqrt{2 - \sqrt{2}}$$
$$= \sqrt{2^2 - \sqrt{2}^2} = \sqrt{4 - 2} = \sqrt{2}$$

تمرين 5 :

أكتب الأعداد التالية على شكل $a\sqrt{b}$:

$$A = 2\sqrt{5} + 5\sqrt{20} = 2\sqrt{5} + 5\sqrt{4 \times 5} = 2\sqrt{5} + 5 \times 2\sqrt{5} = 12\sqrt{5}$$

$$B = \sqrt{45} - \sqrt{5} + 3\sqrt{20} = \sqrt{9 \times 5} - \sqrt{5} + 3\sqrt{4 \times 5}$$
$$= 3\sqrt{5} - \sqrt{5} + 3 \times 2\sqrt{5} = 3\sqrt{5} - \sqrt{5} + 6\sqrt{5} = 8\sqrt{5}$$

$$\begin{aligned} C &= \sqrt{63} + \sqrt{28} - \sqrt{700} = \sqrt{9 \times 7} + \sqrt{4 \times 7} - \sqrt{7 \times 100} \\ &= 3\sqrt{7} + 2\sqrt{7} - 10\sqrt{7} = -5\sqrt{7} \end{aligned}$$

$$\begin{aligned} D &= \sqrt{50} + 3\sqrt{18} - \sqrt{32} = \sqrt{25 \times 2} + 3\sqrt{9 \times 2} - \sqrt{16 \times 2} \\ &= 5\sqrt{2} + 3 \times 3\sqrt{2} - 4\sqrt{2} = 5\sqrt{2} + 9\sqrt{2} - 4\sqrt{2} = 10\sqrt{2} \end{aligned}$$

$$\begin{aligned} E &= -2\sqrt{12} - \sqrt{75} + \sqrt{300} = -2\sqrt{4 \times 3} - \sqrt{25 \times 3} + \sqrt{3 \times 100} \\ &= -4\sqrt{3} - 5\sqrt{3} + 10\sqrt{3} = \sqrt{3} \end{aligned}$$

$$\begin{aligned} F &= 8\sqrt{5} - \sqrt{500} + 4\sqrt{45} = 8\sqrt{5} - \sqrt{5 \times 100} + 4\sqrt{9 \times 5} \\ &= 8\sqrt{5} - 10\sqrt{5} + 12\sqrt{5} = 10\sqrt{5} \end{aligned}$$

$$\begin{aligned} G &= 4\sqrt{24} + 6\sqrt{6} - 2\sqrt{600} = 4\sqrt{4 \times 6} + 6\sqrt{6} - 2\sqrt{6 \times 100} \\ &= 8\sqrt{6} + 6\sqrt{6} - 20\sqrt{6} = -6\sqrt{6} \end{aligned}$$

$$\begin{aligned} H &= \sqrt{44} + \sqrt{99} + 6\sqrt{11} = \sqrt{4 \times 11} + \sqrt{9 \times 11} + 6\sqrt{11} \\ &= 2\sqrt{11} + 3\sqrt{11} + 6\sqrt{11} = 11\sqrt{11} \end{aligned}$$

تمرين 6 :

احذف الجذر من المقام :

$$\frac{5}{\sqrt{3}} = \frac{5 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{5\sqrt{3}}{\sqrt{3}^2} = \frac{5\sqrt{3}}{3}$$

$$\frac{-5}{2\sqrt{3}} = \frac{-5 \times \sqrt{3}}{2\sqrt{3} \times \sqrt{3}} = \frac{-5\sqrt{3}}{2\sqrt{3}^2} = \frac{-5\sqrt{3}}{2 \times 3} = \frac{-5\sqrt{3}}{6}$$

$$\frac{2\sqrt{5}}{\sqrt{3}} = \frac{2\sqrt{5} \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{2\sqrt{15}}{3}$$

$$\frac{1}{2-\sqrt{3}} = \frac{1 \times (2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})} = \frac{2+\sqrt{3}}{2^2-\sqrt{3}^2} = \frac{2+\sqrt{3}}{4-3} = \frac{2+\sqrt{3}}{1} = 2 + \sqrt{3}$$

$$\frac{2}{\sqrt{5}+3} = \frac{2 \times (\sqrt{5}-3)}{(\sqrt{5}+3)(\sqrt{5}-3)} = \frac{2\sqrt{5}-6}{\sqrt{5}^2-3^2} = \frac{2\sqrt{5}-6}{5-9} = \frac{2\sqrt{5}-6}{-4}$$

$$\frac{2\sqrt{3}}{\sqrt{5}+1} = \frac{2\sqrt{3} \times (\sqrt{5}-1)}{(\sqrt{5}+1)(\sqrt{5}-1)} = \frac{2\sqrt{3} \times (\sqrt{5}-1)}{\sqrt{5}^2 - 1^2} = \frac{2\sqrt{3} \times (\sqrt{5}-1)}{5-1} = \frac{2\sqrt{3} \times (\sqrt{5}-1)}{4}$$

$$\frac{\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}} = \frac{(\sqrt{2}+\sqrt{3})}{(\sqrt{2}-\sqrt{3})} \times \frac{(\sqrt{2}+\sqrt{3})}{(\sqrt{2}+\sqrt{3})} = \frac{(\sqrt{2}+\sqrt{3})^2}{\sqrt{2}^2 - \sqrt{3}^2} = \frac{(\sqrt{2}+\sqrt{3})^2}{2-3}$$

$$= \frac{(\sqrt{2}+\sqrt{3})^2}{-1} = -(\sqrt{2} + \sqrt{3})^2$$

$$\frac{1}{1+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{7}} + \frac{1}{\sqrt{7}+\sqrt{9}}$$

$$= \frac{1 \times (1-\sqrt{3})}{(1+\sqrt{3})(1-\sqrt{3})} + \frac{1 \times (\sqrt{3}-\sqrt{5})}{(\sqrt{3}+\sqrt{5})(\sqrt{3}-\sqrt{5})} + \frac{1 \times (\sqrt{5}-\sqrt{7})}{(\sqrt{5}+\sqrt{7})(\sqrt{5}-\sqrt{7})} + \frac{1 \times (\sqrt{7}-\sqrt{9})}{(\sqrt{7}+\sqrt{9})(\sqrt{7}-\sqrt{9})}$$

$$= \frac{1-\sqrt{3}}{1^2-\sqrt{3}^2} + \frac{\sqrt{3}-\sqrt{5}}{\sqrt{3}^2-\sqrt{5}^2} + \frac{\sqrt{5}-\sqrt{7}}{\sqrt{5}^2-\sqrt{7}^2} + \frac{\sqrt{7}-\sqrt{9}}{\sqrt{7}^2-\sqrt{9}^2}$$

$$= \frac{1-\sqrt{3}}{-2} + \frac{\sqrt{3}-\sqrt{5}}{-2} + \frac{\sqrt{5}-\sqrt{7}}{-2} + \frac{\sqrt{7}-\sqrt{9}}{-2}$$

$$= \frac{1-\sqrt{3}+\sqrt{3}-\sqrt{5}+\sqrt{5}-\sqrt{7}+\sqrt{7}-\sqrt{9}}{-2} = \frac{1-3}{-2} = \frac{-2}{-2} = 1$$

تمرين 7 :

حل في IR المعادلات التالية :

$$x^2 - 36 = 0 \quad \Leftrightarrow \quad x^2 - 6^2 = 0 \quad \Leftrightarrow \quad (x - 6)(x + 6) = 0$$

إذن المعادلة تقبل حلين هما : $x = 6$ أو $x = -6$ أو $(x - 6) = 0$ أو $(x + 6) = 0$

$$5 - x^2 = -4 \quad \Leftrightarrow \quad -x^2 = -4 - 5 \quad \Leftrightarrow \quad -x^2 = -9$$

$$x^2 = 9 \quad \Leftrightarrow \quad (x - 9)(x + 9) = 0 \quad \Leftrightarrow \quad (x - 9) = 0 \quad \text{أو} \quad (x + 9) = 0$$

إذن المعادلة تقبل حلين هما : $x = 6$ أو $x = -6$

$$\frac{5}{12}x^2 - \frac{5}{3} = 0 \quad \Leftrightarrow \quad \frac{5}{12}x^2 - \frac{5 \times 4}{3 \times 4} = 0 \quad \Leftrightarrow \quad \frac{5}{12}x^2 - \frac{20}{12} = 0$$

$$\frac{5x^2 - 20}{12} = 0 \quad \Leftrightarrow \quad 5x^2 - 20 = 12 \times 0 \quad \Leftrightarrow \quad 5x^2 - 20 = 0$$

$$5x^2 = 20 \Leftrightarrow x^2 = \frac{20}{5} = 4 \Leftrightarrow x^2 - 4 = 0 \Leftrightarrow x^2 - 2^2 = 0$$

$$(x - 2)(x + 2) = 0 \Leftrightarrow x - 2 = 0 \text{ أو } x + 2 = 0$$

إذن المعادلة تقبل حلين هما : $x = 2$ أو $x = -2$

$$\frac{6+2x^2}{3} - x^2 = 2 \Leftrightarrow \frac{6+2x^2}{3} - \frac{x^2}{1} = \frac{2}{1} \Leftrightarrow \frac{6+2x^2}{3} - \frac{x^2 \times 3}{1 \times 3} = \frac{2 \times 3}{1 \times 3}$$

$$\frac{6 + 2x^2}{3} - \frac{3x^2}{3} = \frac{6}{3} \Leftrightarrow 6 + 2x^2 - 3x^2 = 6 \Leftrightarrow -x^2 = 6 - 6$$

$$-x^2 = 0 \Leftrightarrow x^2 = 0 \Leftrightarrow x = 0 \text{ : إذن المعادلة تقبل حل وحيد هو :}$$

$$9x^2 = 4x^2 \Leftrightarrow 9x^2 - 4x^2 = 0 \Leftrightarrow 5x^2 = 0 \Leftrightarrow x^2 = \frac{0}{5} = 0$$

إذن المعادلة تقبل حل وحيد هو : $x = 0$

$$x^2 + 3 = 0 \Leftrightarrow x^2 = -3 \text{ : إذن المعادلة لا تقبل حل لأن } x^2 \text{ يكون موجب :}$$

سلسلة 3 للجذور المربعة



تمرين 1 :

(1) أحسب مايلي :

$$E = \sqrt{2^2 \times 3^4 \times 10^2}$$

$$F = \sqrt{2^2 + 3^2 + 6^2}$$

$$G = 3\sqrt{3} \times 2\sqrt{12}$$

$$H = \left(\sqrt{\frac{18}{7}} \times \sqrt{\frac{7}{3}} \right)^2$$

$$A = \sqrt{2^2} + \sqrt{5^2}$$

$$B = \sqrt{3\sqrt{16} - 3}$$

$$C = \sqrt{22} \times \frac{\sqrt{2}}{\sqrt{11}}$$

$$D = \sqrt{35} \times \sqrt{\frac{20}{7}}$$

تمرين 2 :

أكتب بدون جذر مربع :

$$F = \sqrt{\sqrt{81}}$$

$$G = \sqrt{2\sqrt{9} + 19}$$

$$H = \frac{6\sqrt{12} + 3\sqrt{27}}{3\sqrt{3}}$$

$$I = \sqrt{\sqrt{100} + 6}$$

$$J = \sqrt{81 - \sqrt{2^2 \times 3^2 + 4^2} - 3}$$

$$A = \sqrt{3} \times \sqrt{15} \times \sqrt{5}$$

$$B = \sqrt{6} \times \sqrt{1,5}$$

$$C = \sqrt{27} \times \sqrt{3}$$

$$D = \sqrt{6} \times \sqrt{\frac{3}{2}}$$

$$E = \frac{\sqrt{28}}{2\sqrt{63}}$$

تمرين 3 :

بسّط مايلي :

$$A = \sqrt{4^2 \times 3^2} + \sqrt{(4+3)^2} - \sqrt{4^2 + 3^2}$$

$$E = 2\sqrt{27} - 5\sqrt{\frac{48}{25}}$$

$$F = \sqrt{5 \times \sqrt{5 \times \sqrt{5 \times \sqrt{5^2}}}}$$

$$B = \sqrt{25\sqrt{16} - \sqrt{\sqrt{16}}}$$

$$C = \sqrt{6 + \sqrt{6} + \sqrt{7} + \sqrt{4}}$$

$$D = \sqrt{25\sqrt{3} \times \sqrt{27}}$$

تمرين 4 :

اجعل المقام عدداً جذرياً :

$$A = \frac{1-\sqrt{2}}{\sqrt{3}} \quad ; \quad B = \frac{2}{\sqrt{\sqrt{3}}} \quad ; \quad C = \frac{\sqrt{3}}{2\sqrt{5}-3\sqrt{2}} \quad ; \quad D = \frac{1}{3-\sqrt{5}} + \frac{1}{3+\sqrt{5}}$$

حل سلسلة 3 للجذور المربعة



تمرين 1:

أحسب مايلي :

$$A = \sqrt{2^2} + \sqrt{5^2} = 2 + 5 = 7$$

$$B = \sqrt{3\sqrt{16} - 3} = \sqrt{3 \times 4 - 3} = \sqrt{12 - 3} = \sqrt{9} = 3$$

$$C = \sqrt{22} \times \frac{\sqrt{2}}{\sqrt{11}} = \sqrt{2} \times \sqrt{11} \times \frac{\sqrt{2}}{\sqrt{11}} = \sqrt{2^2} = 2$$

$$D = \sqrt{35} \times \sqrt{\frac{20}{7}} = \sqrt{5} \times \sqrt{7} \times \frac{\sqrt{4 \times 5}}{\sqrt{7}} = \sqrt{5} \times 2\sqrt{5} = 2 \times \sqrt{5^2} = 2 \times 5 = 10$$

$$E = \sqrt{2^2 \times 3^4 \times 10^2} = \sqrt{2^2} \times \sqrt{(3^2)^2} \times \sqrt{10^2} = 2 \times 9 \times 10 = 180$$

$$F = \sqrt{2^2 + 3^2 + 6^2} = \sqrt{4 + 9 + 36} = \sqrt{49} = 7$$

$$G = 3\sqrt{3} \times 2\sqrt{12} = 6\sqrt{3 \times 12} = 6\sqrt{36} = 6 \times 6 = 36$$

$$H = \left(\sqrt{\frac{18}{7}} \times \sqrt{\frac{7}{3}} \right)^2 = \left(\sqrt{\frac{6 \times 3 \times 7}{7 \times 3}} \right)^2 = \sqrt{6^2} = 6$$

تمرين 2:

أكتب بدون جذر مربع :

$$A = \sqrt{3} \times \sqrt{15} \times \sqrt{5} = \sqrt{15} \times \sqrt{15} = \sqrt{15^2} = 15$$

$$B = \sqrt{6} \times \sqrt{1,5} = \sqrt{6 \times 1,5} = \sqrt{9} = 3$$

$$C = \sqrt{27} \times \sqrt{3} = \sqrt{81} = 9$$

$$D = \sqrt{6} \times \sqrt{\frac{3}{2}} = \sqrt{\frac{6 \times 3}{2}} = \sqrt{\frac{18}{2}} = \sqrt{9} = 3$$

$$E = \frac{\sqrt{28}}{2\sqrt{63}} = \frac{\sqrt{4 \times 7}}{2\sqrt{9 \times 7}} = \frac{2\sqrt{7}}{3 \times 2\sqrt{7}} = \frac{1}{3}$$

$$F = \sqrt{\sqrt{81}} = \sqrt{\sqrt{9^2}} = \sqrt{9} = 3$$

$$G = \sqrt{2\sqrt{9} + 19} = \sqrt{2 \times 3 + 19} = \sqrt{6 + 19} = \sqrt{25} = 5$$

$$H = \frac{6\sqrt{12} + 3\sqrt{27}}{3\sqrt{3}} = \frac{6\sqrt{4 \times 3} + 3\sqrt{9 \times 3}}{3\sqrt{3}} = \frac{6 \times 2\sqrt{3} + 3 \times 3\sqrt{3}}{3\sqrt{3}} = \frac{12\sqrt{3} + 9\sqrt{3}}{3\sqrt{3}} = \frac{21\sqrt{3}}{3\sqrt{3}} = 7$$

$$I = \sqrt{\sqrt{100} + 6} = \sqrt{10 + 6} = \sqrt{16} = 4$$

$$J = \sqrt{81} - \sqrt{2^2 \times 3^2 + 4^2 - 3} = 9 - \sqrt{4 \times 9 + 16 - 3} \\ = 9 - \sqrt{36 + 16 - 3} = 9 - \sqrt{49} = 9 - 7 = 2$$

تمرين 3 :

بسط مايلي :

$$A = \sqrt{4^2 \times 3^2} + \sqrt{(4 + 3)^2} - \sqrt{4^2 + 3^2} = \sqrt{(4 \times 3)^2} + \sqrt{(7)^2} - \sqrt{16 + 9} \\ = \sqrt{(12)^2} + \sqrt{(7)^2} - \sqrt{25} = 12 + 7 - 5 = 14$$

$$B = \sqrt{25\sqrt{16}} - \sqrt{\sqrt{16}} = \sqrt{25 \times 4} - \sqrt{\sqrt{4^2}} = \sqrt{100} - \sqrt{4} = 10 - 2 = 8$$

$$C = \sqrt{6 + \sqrt{6 + \sqrt{7 + \sqrt{4}}}} = \sqrt{6 + \sqrt{6 + \sqrt{7 + 2}}} = \sqrt{6 + \sqrt{6 + \sqrt{9}}} \\ = \sqrt{6 + \sqrt{6 + 3}} = \sqrt{6 + \sqrt{9}} = \sqrt{6 + 3} = \sqrt{9} = 3$$

$$D = \sqrt{25\sqrt{3} \times \sqrt{27}} = \sqrt{25\sqrt{3} \times 27} = \sqrt{25\sqrt{81}} = \sqrt{25 \times 9} \\ = \sqrt{25} \times \sqrt{9} = 5 \times 3 = 15$$

$$E = 2\sqrt{27} - 5\sqrt{\frac{48}{25}} = 2\sqrt{9 \times 3} - 5 \times \frac{\sqrt{16 \times 3}}{5} = 2 \times 3\sqrt{3} - 4\sqrt{3} \\ = 6\sqrt{3} - 4\sqrt{3} = 2\sqrt{3}$$

$$F = \sqrt{5 \times \sqrt{5 \times \sqrt{5 \times \sqrt{5^2}}}} = \sqrt{5 \times \sqrt{5 \times \sqrt{25}}} = \sqrt{5 \times \sqrt{25}} = \sqrt{5 \times 5} = 5$$

تمرين 4 :

اجعل المقام عدداً جذرياً :

$$A = \frac{1-\sqrt{2}}{\sqrt{3}} = \frac{(1-\sqrt{2}) \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{\sqrt{3} \times (1-\sqrt{2})}{\sqrt{3^2}} = \frac{\sqrt{3} \times (1-\sqrt{2})}{3}$$

$$B = \frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{\sqrt{3^2}} = \frac{2\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3} \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{2\sqrt{3} \times \sqrt{3}}{3}$$

$$C = \frac{\sqrt{3}}{2\sqrt{5}-3\sqrt{2}} = \frac{\sqrt{3} \times (2\sqrt{5}+3\sqrt{2})}{(2\sqrt{5}-3\sqrt{2})(2\sqrt{5}+3\sqrt{2})} = \frac{2\sqrt{15}+3\sqrt{6}}{(2\sqrt{5})^2 - (3\sqrt{2})^2} \\ = \frac{2\sqrt{15}+3\sqrt{6}}{20-18} = \frac{2\sqrt{15}+3\sqrt{6}}{2}$$

$$D = \frac{1}{3-\sqrt{5}} + \frac{1}{3+\sqrt{5}} = \frac{1 \times (3+\sqrt{5}) + 1 \times (3-\sqrt{5})}{(3-\sqrt{5})(3+\sqrt{5})} = \frac{3+\sqrt{5}+3-\sqrt{5}}{3^2 - \sqrt{5}^2} = \frac{6}{9-5} = \frac{6}{4}$$